Name _____ Class _____ Date _____

5-5 Practice

Form G

Theorems About Roots of Polynomial Equations

Use the Rational Root Theorem to list all possible rational roots for each equation. Then find any actual rational roots.

1. $x^3 + 5x^2 - 2x - 15 = 0$
   ±1, ±3, ±5, ±15; none
2. $36x^3 + 144x^2 - x - 4 = 0$
   $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{4}{9}, \pm\frac{1}{12}, \pm\frac{1}{18}, \pm\frac{1}{36}$; $-4, -\frac{1}{6}, \frac{1}{6}$
3. $2x^3 + 5x^2 + 4x + 1 = 0$
   $\pm 1, \pm\frac{1}{2}$; $-1, -\frac{1}{2}$
4. $12x^4 + 14x^3 - 5x^2 - 14x - 4 = 0$
   $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$; none
5. $5x^3 - 11x^2 + 7x - 1 = 0$
   $\pm 1, \pm\frac{1}{5}$; $\frac{1}{5}$, 1
6. $x^3 + 81x^2 - 49x - 49 = 0$
   ±1, ±7, ±49; none

A polynomial function $P(x)$ with rational coefficients has the given roots. Find two additional roots of $P(x) = 0$.

7. $2 + 3i$ and $\sqrt{7}$ $2 - 3i, -\sqrt{7}$
8. $3 - \sqrt{2}$ and $1 + \sqrt{3}$ $3 + \sqrt{2}, 1 - \sqrt{3}$
9. $-4i$ and $6 - i$ $4i, 6 + i$
10. $5 - \sqrt{6}$ and $-2 + \sqrt{10}$ $5 + \sqrt{6}, -2 - \sqrt{10}$
11. $\sqrt{5}$ and $-\sqrt{13}$ $-\sqrt{5}$ and $\sqrt{13}$
12. $1 - \sqrt{10}$ and $2 + \sqrt{2}$ $1 + \sqrt{10}$ and $2 - \sqrt{2}$

Write a polynomial function with rational coefficients so that $P(x) = 0$ has the given roots.

13. 4 and 6 $P(x) = x^2 - 10x + 24$
14. -5 and -1 $P(x) = x^2 + 6x + 5$
15. $3i$ and $\sqrt{6}$ $P(x) = x^4 + 3x^2 - 54$
16. $2 + i$ and $1 - \sqrt{5}$
    $P(x) = x^4 - 6x^3 + 9x^2 + 6x - 20$
17. -5 and $3i$ $P(x) = x^3 + 5x^2 + 9x + 45$
18. i and $5i$ $P(x) = x^4 + 26x^2 + 25$

What does Descartes' Rule of Signs say about the number of positive real roots and negative real roots for each polynomial function?

19. $P(x) = 3x^3 + x^2 - 8x - 12$ 1 positive real root; 2 or 0 negative real roots
20. $P(x) = 2x^4 - x^3 - 3x + 7$ 2 or 0 positive real roots; 0 negative real roots
21. $P(x) = 4x^5 - x^4 - x^3 + 6x^2 - 5$ 3 or 1 positive real roots; 2 or 0 negative real roots
22. $P(x) = x^3 + 4x^2 + x - 6$ 1 positive real root; 2 or 0 negative real roots

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5-5 Practice (continued) *Form G*

Theorems About Roots of Polynomial Equations

Find all rational roots for $P(x) = 0$.

23. $P(x) = x^3 - 5x^2 + 2x + 8$ **4, 2, −1**

24. $P(x) = x^3 + x^2 - 17x + 15$ **3, 1, −5**

25. $P(x) = 2x^3 + 13x^2 + 17x - 12$ **$-4, -3, \frac{1}{2}$**

26. $P(x) = x^3 - x^2 - 34x - 56$ **7, −2, −4**

27. $P(x) = x^3 - 18x + 27$ **3**

28. $P(x) = x^4 - 5x^2 + 4$ **−2, −1, 1, 2**

29. $P(x) = x^3 - 6x^2 + 13x - 10$ **2**

30. $P(x) = x^3 - 5x^2 + 4x + 10$ **−1**

31. $P(x) = x^3 - 5x^2 + 17x - 13$ **1**

32. $P(x) = x^3 + x + 10$ **−2**

33. $P(x) = x^3 - 5x^2 - x + 5$ **1, −1, 5**

34. $P(x) = x^3 - 12x + 16$ **−4, 2**

35. $P(x) = x^3 - 2x^2 - 5x + 6$ **−2, 1, 3**

36. $P(x) = x^3 - 8x^2 - 200$ **10**

37. $P(x) = x^3 + x^2 - 5x + 3$ **1, −3**

38. $P(x) = 4x^3 - 12x^2 - x + 3$ **$3, \frac{1}{2}, -\frac{1}{2}$**

39. $P(x) = x^3 + x^2 - 7x + 2$ **2**

40. $P(x) = 12x^3 + 31x^2 - 17x - 6$ **$-3, \frac{2}{3}, -\frac{1}{4}$**

Write a polynomial function $P(x)$ with rational coefficients so that $P(x) = 0$ has the given roots.

41. $\sqrt{3}, 2, -i$
$P(x) = x^5 - 2x^4 - 2x^3 + 4x^2 - 3x + 6$

42. $5, 2i$
$P(x) = x^3 - 5x^2 + 4x - 20$

43. $-1, 3 + i$
$P(x) = x^3 - 5x^2 + 4x + 10$

44. $-\sqrt{7}, i$
$P(x) = x^4 - 6x^2 - 7$

45. $-4, 4i$
$P(x) = x^3 + 4x^2 + 16x + 64$

46. $6, 3 - 2i$
$P(x) = x^3 - 12x^2 + 49x - 78$

47. Error Analysis A student claims that $2i$ is the only imaginary root of a polynomial equation that has real coefficients. Explain the student's mistake.
The student forgot the conjugate imaginary root $-2i$.

48. You are building a rectangular sandbox for a children's playground. The width of the sandbox is 4 times its height. The length of the sandbox is 8 ft more than 2 times its height. You have 40 ft^3 of sand available to fill this sandbox. What are the dimensions of the sandbox? **height = 1 ft, width = 4 ft, length =10 ft**

49. Writing According to the Rational Root Theorem, what is the relationship between the polynomial equation $2x^4 - x^3 - 7x^2 + 5x + 3 = 0$ and rational roots of the form $\frac{p}{q}$, where $\frac{p}{q}$ is in simplest form?
***p* must be a factor of 3 and *q* must be a factor of 2.**