ABCALC Apps of Derivatives Review Session Solutions

CONCEPTS:

1. When looking for absolute extrema, where do the possible extrema exist, and how do you find them?

   possible extrema exist at endpts and critical pts ($f' = 0$ or $f' =$ und)
   plug-in all of these to find absolute extrema.

2. How do you justify relative extrema?

   f' changes from + to − for max OR $f' = 0$/und and $f'' > 0$ min
   f' changes from − to + for min $f' = 0$/und and $f'' < 0$ max

3. How do you justify that a function is increasing or decreasing?

   Inc → $f' > 0$ dec: $f' < 0$

4. How do you justify that a function is concave up or concave down?

   concave down: $f'' < 0$ concave up: $f'' > 0$

5. How do you justify that a function has a point of inflection? f'' changes sign.

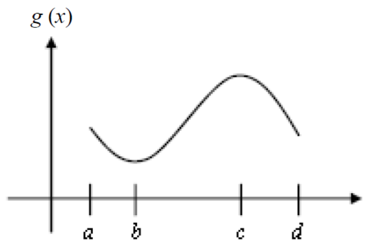
6. Using the graph of $g(x)$ below, determine the signs of $g'(x)$ and $g''(x)$ at each point. Explain your reasoning.

At $x = a$ …

At $x = b$ …

At $x = c$ …

At $x = d$ …



x	$g'(x)$	reason	$g''(x)$	reason
a	−	g(x) is decreasing	+	g(x) is concave up
b	0	horizontal tan line	+	g(x) is concave up
c	0	horiz. tan line	−	g(x) is concave down
d	−	g(x) decreasing	−	g(x) is concave down

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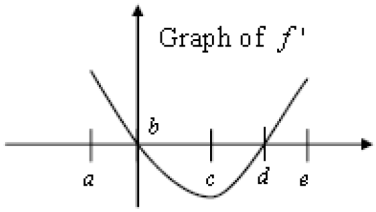
7. Given the graph of f' below answer each of the following questions, and justify your response with a statement that contains the phrase "since f' ________________ …"



a) When is f increasing?

$(a,b) \cup (d,e)$

$f'>0$

b) When is f decreasing?

(b,d) $f'<0$

c) When is f concave up?

(c,e) f' is increasing

d) When is f concave down?

(a,c) f' is decreasing

e) When does f have a relative maximum?

b since f' changes from + to −

f) When does f have a relative minimum?

d f' changes from − to +

g) When does f have a point of inflection?

c f' changes from dec to inc

Find the value of c guaranteed by the MVT for $f(x) = 4x^2 + 5x$ on the interval [–2, 1].

↑ cont. and diff over [-2,1]

$$\frac{f(1)-f(-2)}{1--2} = f'(x)$$

$$\frac{9-6}{3} = 1$$

$$1 = 8x+5$$

$$-4 = 8x$$

$$\boxed{x = -\frac{1}{2}}$$

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Suppose $y = x^3 - 3x$. [No Calculator]

a) Find the zeros of the function.

$x(x^2-3)=0 \quad x=0 \quad x=\sqrt{3} \quad x=-\sqrt{3}$

b) Determine where y is increasing or decreasing and justify your response.

$y' = 3x^2 - 3 \qquad 3x^2 = 3$

$0 = 3x^2 - 3 \qquad x^2 = 1$

$x = \pm 1$

$+ \quad - \quad +$

$\nearrow -1 \searrow 1 \nearrow$

inc: $(-\infty,-1)\cup(1,\infty)$ $f'(x)>0$

dec: $(-1,1)$ $f'(x)<0$

c) Determine all local extrema and justify your response.

min at $x=1$ since f' changes from $-$ to $+$

max at $x=-1$ since f' changes from $+$ to $-$

d) Determine the points where y is concave up or concave down, and find any points of inflection. Justify your responses.

POI at $x=0$ since f'' changes sign

$y'' = 6x \qquad x=0$

$- \quad +$

$\cap \; 0 \; \cup$

concave down: $(-\infty,0)$ $f''<0$

concave up: $(0,\infty)$ $f''>0$

e) Use all your information to sketch a graph of this function.

The function f is continuous on [0, 3] and satisfies the following:

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3
f	0	Neg	–2	Neg	0	Pos	3
f'	–3	Neg	0	Pos	DNE	Pos	4
f''	0	Pos	1	Pos	DNE	Pos	0

a) Find the absolute extrema of f and where they occur.

f': $- \quad + \quad +$

f: $\searrow 1 \nearrow 2 \nearrow$

cusp place

f'': $+ \quad +$

f: $\cup \; 2 \; \cup$

$f(0)=0$

$f(1)=-2$

$f(2)=0$

$f(3)=3$

Abs max of 3 at $x=3$

Abs min of -2 at $x=1$

b) Find any points of inflection.

No pts of inflection since f'' does not change sign.

c) Sketch a possible graph of f.